MATH 141: Quiz 3

Name: key

Directions:

* Show your thought process (commonly said as "show your work") when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

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1. A function $f(x)$ is continuous at $x = -3$. Using the mathematical definition of continuity, state the three conditions that must be true.

① $f(-3)$ is defined

② $\lim_{x \to -3} f(x)$ exists

③ $\lim_{x \to -3} f(x) = f(-3)$

2. **Using the definition of continuity,** determine whether the following function is continuous at $x = 0$:

$$f(x) = \begin{cases} (x-1)^2 & x > 0 \\ 0 & x = 0 \\ (x+1)^2 & x < 0 \end{cases}$$

① $f(0) = 0$ ✓

② $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x-1)^2 \overset{\text{limit laws}}{=} \left(\lim_{x \to 0^+} x - \lim_{x \to 0^+} 1 \right)^2 = (0-1)^2 = 1$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x+1)^2 \underset{\text{limit laws}}{=} \left(\lim_{x \to 0^-} x + \lim_{x \to 0^-} \right)^2 = (0+1)^2 = 1$

$\therefore \lim_{x \to 0} f(x) = 1$ ✓

③ $\lim_{x \to 0} f(x) = 1 \neq 0 = f(0)$

$\boxed{\therefore f(x) \text{ is not continuous at } x = -3}$

3. State in interval notation where this function is continuous:

$$f(x) = \frac{\sin(x^2+1)}{\boxed{2x^2-5x+2}} - \sqrt{2x-2}$$

Find domain:

① Problems:

ⓐ division by 0.

$2\ \ -1$
$1\ \ -2$

$$2x^2 - 5x + 2 = 0$$

$$(2x-1)(x-2) = 0$$

$$2x - 1 = 0 \qquad x - 2 = 0$$

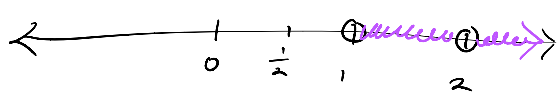
$$\boxed{x = \tfrac{1}{2}, \quad x = 2}$$

ⓑ Square root of negative.

$$2x - 2 < 0$$

$$\boxed{x < 1}$$

② Remove problems from $\mathbb{R}$



Domain: $(1, 2) \cup (2, \infty)$

Because this function is continuous on its domain,

$f(x)$ is continuous on $(1, 2) \cup (2, \infty)$.